

Energy and Delivery Capacity of Wireless Sensor Networks with Random Duty-Cycles

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Abstract¹ – Since battery energy is often a premium resource in wireless sensor networks, many systems attempt to conserve battery energy by turning the transceivers off in a process referred to as duty-cycling. In [4-5], the authors proposed two different random duty-cycling protocols utilizing pseudo random number generators to efficiently exchange ON/OFF schedules among neighboring nodes for transmission coordination. We propose two simple transmission schemes that are superior to the performance of the schemes in [4-5] in terms of energy consumption and a macro network metric termed delivery capacity. The delivery capacity measures the maximum long term average unicast traffic delivery capability of a network. Through analyses and simulations, we present a detailed energy and delivery capacity tradeoff study of the transmission schemes in [4-5] and the schemes we propose. We show that one of our proposed schemes is near-optimal in energy consumption for low traffic loads. In addition, we provide insights on the appropriate ON/OFF duty-cycling parameters that can achieve the optimal energy and delivery capacity tradeoff curve.

Keywords – wireless sensor network, duty-cycle, energy, delivery capacity

I. INTRODUCTION

In recent years, there has been a prolific growth in the research and development of large-scale wireless sensor networks (WSN), spurred by a confluence of advancements in diverse technical fields such as embedded processing, signal processing, and networking. Such WSNs enable distributed sensing in diverse geographical areas such as college campuses, farms, forests, and seas, thereby significantly extending the traditionally available information gathering capabilities. Comparatively speaking, battery technology has not experienced advances commensurate with the rate of other technological developments. As a result, battery energy is often a premium resource in WSNs. Since most radio frequency (RF) transceivers consume a significant amount of battery energy when they are *ON*, many WSN protocols attempt to conserve battery energy by turning the transceivers *OFF* in a process referred to as *duty-cycling*. The duty-cycles may be periodic with fixed ON/OFF schedules, random with randomly generated and time-varying ON/OFF schedules, or coordinated among neighboring nodes depending on traffic demand [1-5]. In the implementation of energy conserving duty-cycling

WSNs, slotted multiple-access control (MAC) schemes which enforce communication on synchronized slot boundaries have gained popularity [4, 6]. Such synchronized duty-cycling schemes can realize significant network energy savings [4]².

Since the wireless channel is a shared medium, simultaneous transmissions from multiple neighboring nodes may result in packet collisions, rendering the received information indiscernible. To avoid energy wastage, a node should ideally turn on to receive in a slot only if there is a transmission intended for it and if the transmitted packets can be received successfully without collision. By the same token, ideally, a node should only transmit to a neighbor if the latter can receive the packets successfully. This requires coordination of ON/OFF schedules among network nodes³.

Since the optimal time-division multiple access (TDMA) schedule is often difficult to compute [7], researchers have proposed random duty-cycling protocols where nodes achieve some degree of coordination by sharing information about each others' pseudo random number generators (pRNG) [4-5, 8]. By such an information exchange, neighboring nodes can achieve loose coordination among them – the schedules are themselves random in nature but by exchanging a small set of pRNG parameter values (such as *seed* and *cycle position*) each node can construct the exact ON/OFF state information of all of its neighbors in a large number of subsequent slots. In [4], the authors propose to reduce the amount of wasted transmission energy by transmitting to a neighboring node only if the node is ON and receiving in a slot. In [5], the authors propose exchanging the schedule among all two-hop neighbors with transmitters competing for a neighboring receiver's slot in a process similar to slotted-ALOHA.

Although the method proposed in [4] and an improved version of [5]⁴ are more energy efficient than a purely random transmission scheme without schedule exchanges, they do not capture all the realizable energy savings. In this paper, we compare six different schemes including [4] and an improved version of [5]. We show by means of stochastic analysis and simulation that it is possible to achieve significantly higher

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² This requires slot synchronization, which can be achieved via GPS or by techniques proposed in [9].

³ In a system where ON/OFF schedules of neighboring nodes are uncoordinated, the total throughput can be significantly limited and the energy consumption can be much higher than necessary. In principle, if traffic pattern is known in advance, optimal TDMA schedules can achieve the highest throughput with minimum energy consumption.

⁴ The method proposed in [5] was not meant for saving energy but for reducing collisions to attain higher throughput. The focus of [4] was on achieving energy savings without much attention to achievable throughput.

energy savings by incorporating receiver energy saving strategies. This is intuitive since in well connected WSNs, a single transmission, which is a wireless broadcast, could result in significant energy expenditure due to reception at several nodes. Hence, intelligently turning off receivers which will otherwise be idle, suffer a collision, or receive packets not intended for it has significant payoffs in terms of energy savings without sacrificing bits delivered. We propose two schemes which adopt the receiver energy saving strategy along with other simple techniques and show that both schemes outperform [4-5] in terms of energy savings as well as total bits delivered in a slot. These schemes are described in detail in Section II.

We propose and use a macro network metric termed *delivery capacity* to better contrast the spatial reuse and energy saving features of the different schemes. This metric measures the maximum number (long term average) of successful packet receptions in a unit slot time given a particular scheme. Hence the delivery capacity is an intrinsic property of the network topology and the particular transmission scheme used. In our study, we assume that each node transmits to every one of its neighbors with finite probability. The scheme that yields higher delivery capacity achieves higher spatial reuse given a network topology, hence is able to deliver more bits successfully in a slot. Therefore, the ideal transmission scheme should achieve high delivery capacity while balancing energy consumptions.

Through analyses and simulations, we present a detailed comparative energy and delivery capability study of six transmission schemes under uni-cast and broadcast traffic as well as provide insights on the appropriate ON/OFF duty-cycling parameters that can achieve the optimal energy and delivery capability tradeoff curve. In addition to performance gains over existing schemes, we show in Section V that one of our proposed schemes can yield near-optimal energy savings for low traffic loads. The simplicity of this scheme, combined with near-optimal energy performance makes it an ideal message delivery scheme for WSNs with low traffic loads.

This paper is organized as follows. Section II presents the models we use in our analyses and simulations. Sections III and IV present analytical and simulation results respectively. Section V presents a discussion of our proposed schemes. Section VI summarizes the key results of this paper.

II. MESSAGE DELIVERY SCHEMES AND METRICS

Consider a network with N nodes, each with a transmission radius of r . We adopt a distance-based link model in that a link exists between two nodes if the inter-node distance is less than r . The resulting network can be represented by a graph $G(V, E)$, where V is the set of vertices representing the nodes and E is the set of edges representing the links. We assume the graph is connected such that there exists at least one path connecting any two node pairs. The nodes are arbitrarily indexed from 1 to N . Each node i has a set of neighboring nodes denoted by H_i . The cardinality of H_i is denoted by h_i , where $h_i \in [1, N - 1]$.

We assume that time is divided into equal duration slots. Slot synchronization among nodes can be achieved using techniques in [9]. In each time slot, a node can be in one of three states: ON-RX, ON-TX, and OFF. In the ON-RX state, a node turns on its reception modules to receive incoming traffic. In the ON-TX state, a node turns on its transmission modules to transmit outgoing traffic. The transmission may be intended for one of the neighboring nodes (*uni-cast hop traffic*) or for multiple neighboring nodes (*multicast or broadcast hop traffic*). Each transmission is omnidirectional and is heard by all of the neighboring nodes in the ON-RX state. We assume that when a node chooses a neighboring node to transmit to, there is always enough packets to send. Finally, in the OFF state, a node turns off all of its communication modules.

In each slot, each node chooses its state independently of each other and independent of previous states with probabilities p_{rx} , p_{tx} and $p_{off} = 1 - p_{rx} - p_{tx}$ for states ON-RX, ON-TX, and OFF respectively, where $0 < p_{rx}, p_{tx} < 1$ and $0 \leq p_{off} < 1$.

For practical implementation of the random duty-cycling protocols, the state probabilities may be different for each node depending on the local traffic load and topology. In the subsequent analysis we assume that the state probabilities are identical at each node for simplicity and for gaining some insights into the dependency of network performance on the state probabilities. Tuning state probabilities at each node for local performance improvement is subject to future studies.

A. Performance Metrics

In a WSN, energy consumption and the amount of data traffic the network can sustain are two important metrics. Since a wireless network is inherently interference limited, the amount of sustainable traffic is limited; hence transmission strategies and routing schemes that can best utilize the spatial reuse feature while limiting the amount of pass-through traffic tend to increase the amount of end-to-end traffic the network can sustain. Assuming that each node has some traffic (either new or pass-through) to send to each of its neighboring nodes, and given a network topology and state probabilities, we would like to design message delivery schemes that achieve high spatial reuse while attaining low energy consumption (ie. limiting the amount of wasted energy due to collision or nodes turning on but are idle).

We define two performance metrics that capture the intrinsic spatial reuse and message delivery capabilities of a network given the network topology and message delivery schemes we will describe shortly: reception success and hop-delivery success. A node i receives a transmission successfully (*reception success*) in a slot if it is in the ON-RX state and exactly one of its neighbors is in the ON-TX state (ie. no collision at the receiver). This is a stringent collision model that does not account for the possibility of capture due to received power differences [10]; hence the number of successes in the subsequent analyses is conservative compared to what may be achievable. Note that a reception success by node i does not mean that node i is the intended recipient of the message received. We define *hop-delivery success* to be a re-

ception success at the intended recipient. Here we are only referring to delivery over one-hop and not end-to-end delivery.

We define *Reception Capacity* and *Delivery Capacity* to be the maximum average reception success and average hop-delivery success for a given network topology and message delivery scheme, where the average is taken over a large number of slots. *Ideally, we would like to attain a high delivery capacity for a given message delivery scheme while keeping the energy consumption of the network low.*

From an energy perspective, since collisions may occur in the network, opportunistically turning nodes off can generate significant energy savings. Furthermore, if the amount of traffic load in the network is much lower than the maximum sustainable traffic, then turning nodes off through a duty-cycling protocol can further reduce the energy consumption of the network. We adopt a simple energy model in our study. In each slot, a node expends E_{tx} , E_{rx} , and 0 amounts of energy if it is in the ON-TX, ON-RX, and OFF states respectively. The average amount of energy consumed by the network in a slot is then $E_{tx}\bar{N}_{tx} + E_{rx}\bar{N}_{rx}$, where \bar{N}_{tx} and \bar{N}_{rx} are the average number of nodes in the ON-TX and ON-RX states in a slot respectively. This model assumes that the transmission power level is the same regardless of the distance between two nodes. We suppress the possibility of power control here so that the effects due to the various message delivery schemes can be highlighted.

B. Random Schedule and Energy Saving Strategies

Now we describe six random message delivery schemes. Each scheme has unique features that allow us to explore the incremental gains of various parameters and strategies that improve total energy consumption and hop-delivery success. The first scheme (S1) is the simplest but the most underperforming. This serves as a baseline for comparison. The second scheme (S2) is used in [4] and the fifth scheme (S5) is similar to the scheme proposed in [5]. We propose three additional schemes (S3, S4, and S6) and perform detailed performance comparisons of the six schemes. These schemes are summarized in Table 1 in terms of their energy saving and hop-delivery success improvement strategies.

S1. Random Scheme without Schedule Exchange

Under this scheme, nodes do not exchange state information. In each slot, nodes independently choose states probabilistically as described. Suppose node i is in the ON-TX state in a slot. For unicast traffic, i randomly selects one of the nodes j , where $j \in H_i$, and transmits packets for j . For broadcast traffic, i simply transmits.

For a node i that transmits, there are several possibilities:

T1: there does not exist a node in the ON-RX state in H_i - the transmission is unsuccessful.

T2: there exists at least one node in the ON-RX state in H_i . In this case, there are several possibilities for the nodes in H_i under unicast traffic:

R1: the intended receiver j is in the OFF state.

R2: the intended receiver j is in the ON-RX state and receives the packets. This counts as one hop-delivery success.

R3: the intended receiver j is in the ON-RX state but receives collided signals

R4: the packets are received by a node(s) other than j and are discarded.

Additionally, for a node k in the ON-RX state, it is possible that:

R5: there does not exist a node in the ON-TX state in H_k . Under broadcast traffic, the subset of nodes in H_i that are in the ON-RX states and do not themselves have other neighboring nodes in the ON-TX state can receive the packets successfully. Each node that receives the packets successfully is counted as one reception success.

S2. One-Hop Schedule Exchange

In each slot, the state of each node is determined by a pRNG. At the beginning of network operation, each node informs its neighbors of its pRNG's seed and cycle position. By such an exchange, each node can compute the state of all its neighbors in every slot. This technique is described in more detail in [4-5, 8].

Having attained neighboring nodes' ON-TX/ON-RX/OFF schedules, a node i in the ON-TX state randomly selects one of the nodes j , where $j \in H_i$ and j is in the ON-RX state to transmit to. Under this scheme, scenarios T1-T2 and R2-R5 may occur while R1 is guaranteed not to occur. Essentially, with one-hop schedule exchange, a transmitter can improve the probability of transmission success by transmitting to a neighboring node that is not in the ON-TX or OFF states.

S3. One-hop Schedule Exchange & Nodes Turn OFF

Under this scheme, we incorporate opportunistic energy saving strategies. In addition to S2, if a node m in the ON-TX state does not have any neighbor in the ON-RX state in a slot, then node m modifies its state to OFF in that slot. Similarly if a node k in the ON-RX state does not have any neighbor in the ON-TX state in a slot, then node k modifies its state to OFF in that slot. In addition, a node k in the ON-RX state modifies its state to OFF in a slot if more than one of its neighbors is in the ON-TX state. The state modification for each slot can be done all at once after obtaining neighbors' pRNG seed and cycle position information. Here T2, R1, R2, and R4 may occur while T1, R3, and R5 do not. Note that R1 occurs here because of the state modification from ON-RX to OFF (these nodes would not have received packets anyway due to collision). With this scheme, the nodes that would have experienced collisions or idle slots are now opportunistically turned OFF to save energy.

S4. Two-hop Schedule Exchange & Nodes Turn OFF

Under this scheme, the state for each node is determined by a pRNG as in S2. Each node exchanges the seed and cycle position of its pRNG and those of its neighbors to all of its neighbors. By such a two-hop exchange, each node knows the state of all of its two-hop neighbors in every slot.

TABLE I. Summary of Hop-Delivery Improvement and Energy Saving Strategies

Scheme	Hop-Delivery	Energy	Schedule Exchange
S1	--	--	--
S2	- Select a neighbor that is in the ON-RX state	--	One-hop
S3	- Same as S2	- Modify nodes in the ON-TX state to OFF if there is no neighbor in the ON-RX state - Modify nodes in the ON-RX state to OFF if there does not exist exactly one neighbor in the ON-TX state	One-hop
S4	- Select a neighbor in the ON-RX state that will not experience collision	- Same as S3 - Modify nodes in the ON-TX state to OFF if there does not exist at least one neighbor that will not experience collision	Two-hop
S5	- Same as S2 - Utilize random backoff with probability of transmission that depends on the number of nodes in the ON-TX state in a receiver node's neighborhood	- Same as S3 - Modify nodes in the ON-TX state to OFF if the nodes do not transmit according to the backoff probability	Two-hop
S6	- Same as S4 - If there does not exist a neighbor in the ON-RX state that will not experience collision, randomly choose a neighbor in the ON-RX state. Utilize random backoff with probability of transmission that depends on the number of nodes in the ON-TX state in the receiver node's neighborhood	- Same as S5 (nodes in the ON-TX state do not modify their states to OFF if there exists at least one neighbor in the ON-RX state that will not experience collision)	Two-hop

In each slot, a node opportunistically modifies its state to OFF as in S3. In addition, since a node has two-hop schedule information, it knows the modified schedule of all of its neighbors. After determining the new schedule, a node modifies its state from ON-TX to OFF if there are no neighbors in the ON-RX state in that slot. This saves even more energy over S3. In addition, a node i in the ON-TX state randomly select one of the nodes j , where $j \in H_i$ and j is in the ON-RX state under the new schedule to transmit to. This increases the probability of transmission success as the transmitted packets are guaranteed to be received successfully by the intended recipient. Here T2, R2, and R4 may occur while T1, R1, R3, and R5 do not.

S5. Modified Seedex

Under this scheme, each node exchanges two-hop schedule information as in S4. In each slot, a node i in the ON-TX state selects one of its neighbors j as a potential receiver, where $j \in H_i$ and j is in the ON-RX state. The actual transmission is dictated by a probability p that is inversely proportional to the number of nodes in H_j that are in the ON-TX state. For fairness in the subsequent comparisons, we add some energy saving strategies: (1) Nodes that do not transmit (due to probabilistic backoff) are modified to the OFF state, (2) nodes modify their states from ON-RX and ON-TX to OFF if there are no nodes in the ON-TX or ON-RX state respectively in a node's neighborhood. Here T2, R2, R3, and R4 may occur while T1, R1, and R5 do not.

S6. Combination of S4 and S5

This scheme combines features of S4 and S5. First, nodes are modified from ON-RX and ON-TX to OFF if there are no nodes in the ON-TX or ON-RX state respectively in a node's neighborhood. In each slot, a node i in the ON-TX state randomly selects one of the node j to transmit to, where $j \in H_i$, j is in the ON-RX state, and i is the only node in H_j that is in the ON-TX state. If no such node j exists in H_i , then node

i randomly selects one of its neighboring nodes k as a potential receiver, where $k \in H_i$ and k is in the ON-RX state. The actual transmission is dictated by a probability p that is inversely proportional to the number of nodes in H_k that are in the ON-TX state. Nodes that do not transmit are modified to the OFF state.

Here nodes that can receive with guaranteed success are given priority while transmitters in the neighborhood of nodes that may potentially experience collision compete for the slot. Under this scheme, T1, T2, and R1-R4 may occur while R5 does not.

III. ENERGY AND DELIVERY CAPACITY ANALYSIS

We compare the message delivery schemes S1-S6 in terms of the following performance metrics: average energy consumption, reception success, hop-delivery success, reception capacity, and delivery capacity.

By the random state selection process in each slot, the number of nodes in each of the states (prior to nodes modifying their states) follows a multinomial distribution

$$\Pr(N_{tx} = x_1, N_{rx} = x_2, N_{off} = x_3) = \frac{N!}{\prod_{i=1}^3 x_i!} p_{tx}^{x_1} p_{rx}^{x_2} (1 - p_{tx} - p_{rx})^{x_3}$$

where $\sum_{i=1}^3 x_i = N$ and N_{tx} , N_{rx} , and N_{off} are the number of nodes in ON-TX, ON-RX, and OFF states respectively.

Theorem 1. For a given network topology and state probabilities p_{tx} and p_{rx} , the expected energy consumption in a slot under message delivery schemes S1-S4 satisfy

$$\bar{E}_{S4} < \bar{E}_{S3} < \bar{E}_{S2} = \bar{E}_{S1}$$

Proof: The expected number of nodes in the ON-TX, ON-RX, and OFF states respectively under the various schemes are:

$$S1: p_{tx}N, p_{rx}N, \text{ and } N(1 - p_{tx} - p_{rx})$$

$$S2: p_{tx}N, p_{rx}N, \text{ and } N(1 - p_{tx} - p_{rx})$$

$$S3: p_{tx}(N - u), p_{rx}v, \text{ and } N(1 - p_{tx}) + p_{tx}u - p_{rx}v$$

S4: $p_{tx}(N-u)-\varepsilon$, $p_{rx}v$, and $N(1-p_{tx})+p_{tx}u-p_{rx}v+\varepsilon$, where $u = \sum_{i=1}^N (1-p_{rx})^{h_i}$, $v = \sum_{i=1}^N h_i p_{tx} (1-p_{tx})^{h_i-1}$, and $\varepsilon > 0$. The exact expressions for ε is difficult to derive analytically since it depends on two hop neighbor topologies. However, we note that ε represents the expected number of nodes modifying their states from ON-TX to OFF due to neighboring nodes intelligently modifying their states from ON-RX to OFF. There is a finite probability for this to occur in any topology and for any state probabilities. Hence the expected number of nodes in the ON-TX state under S4 is less than under S3. Since $(1-p_{rx})^{h_i} > 0, \forall p_{rx}, h_i$ and $h_i p_{tx} (1-p_{tx})^{h_i-1} < 1, \forall p_{tx}, h_i$, we obtain $p_{tx}N > p_{tx}(N-u)$ and $p_{rx}N > p_{rx}v$, which establishes the results. ■

Under schemes S5 and S6, the expected number of nodes in the ON-RX states is $p_{rx}(N-w)$, where $w = \sum_{i=1}^N (1-p_{tx})^{h_i}$. One can show that $p_{rx}v < p_{rx}(N-w)$. Hence, given a topology and state probabilities, S4 has the smallest expected number of nodes in the ON-RX state. The transmission probability of a node under schemes S5 and S6 has long range dependencies on the network topology (ie. the probability of transmission of a node depends on the transmission probabilities of its neighbors, which themselves are dependent on their neighbors, etc.); hence analytical expressions for the expected number of nodes in the ON-TX state are difficult to derive, even for simple topologies such as a line network.

Now we show that for a particular hop-delivery success, scheme S4 yields the minimum number of transmissions. Furthermore, each transmission is guaranteed to be successful. Define ψ to be an N -tuple of node states $[\psi_1, \dots, \psi_N]$, where $\psi_i \in \{\text{ON-TX}, \text{ON-RX}, \text{OFF}\}$ for node i in a slot. For a given network topology, consider the set, Ω , of all possible 3^N N -tuple of node states. After the application of a message delivery scheme S_k , where $k=1, \dots, 6$, let Ω^{S_k} represent the resulting set of all possible N -tuple of node states, where $\Omega^{S_k} \subseteq \Omega$. For each $\psi' \in \Omega^{S_k}$, a particular hop-delivery success is attained. Let the number of nodes in the ON-TX, ON-RX, and OFF states be $N_{tx}^{S_k}(\psi')$, $N_{rx}^{S_k}(\psi')$, and $N_{off}^{S_k}(\psi')$, where $N_{tx}^{S_k}(\psi') + N_{rx}^{S_k}(\psi') + N_{off}^{S_k}(\psi') = N$.

Theorem 2. For a given hop-delivery success, $N_{hop-delivery}$, in a slot, the number of nodes in the ON-TX and ON-RX states, $(N_{tx}^{S_k}(\psi'), N_{rx}^{S_k}(\psi')), \forall \psi' \in \Omega^{S_k} | N_{hop-delivery}$ is achieved, and $\forall k$, satisfies $N_{hop-delivery} \leq \min(N_{tx}^{S_k}(\psi'), N_{rx}^{S_k}(\psi'))$. In particular, $N_{hop-delivery} = N_{tx}^{S_4}(\psi') \leq N_{rx}^{S_4}(\psi'), \forall \psi' \in \Omega^{S_4} | N_{hop-delivery}$ is achieved
Proof: Under uni-cast traffic, each node in the ON-TX or ON-RX state can attain at most one hop-delivery success. Hence, for a given network topology and a hop-delivery success

$N_{hop-delivery}$, $N_{hop-delivery} \leq \min(N_{tx}^{S_k}(\psi'), N_{rx}^{S_k}(\psi'))$ is satisfied for the conditions given. Under scheme S4, each node in the ON-TX state transmits to a neighboring node that is the intended recipient and is guaranteed to receive the packets successfully. Hence, each transmission corresponds to a hop-delivery success. Together,

$N_{hop-delivery} = N_{tx}^{S_4}(\psi') \leq N_{rx}^{S_4}(\psi'), \forall \psi' \in \Omega^{S_4} | N_{hop-delivery}$ is achieved is established. This implies $N_{tx}^{S_4} \leq \min(N_{tx}^{S_k}(\psi'), N_{rx}^{S_k}(\psi')), \forall \psi' \in \Omega^{S_k} | N_{hop-delivery}$ is achieved, for $k=1, 2, 3, 5, 6$. ■

We now derive an upperbound on the average reception success under schemes S1-S4. The Seedex scheme (S5) does not address multicast or broadcast hop traffic, hence S5 and S6 are not included in this analysis.

Theorem 3. The expected number of reception successes for schemes S1-S4 in a slot is upperbounded by $N/e \approx 0.37N$ for large N , where e is the familiar transcendental number. We term this upperbound *Reception Capacity Bound*.

Proof: Recall that a reception success occurs if a node successfully receives some packets (regardless of whether or not the node is the intended recipient of the packets; alternatively, the traffic could be directed towards all neighboring nodes). In this regard, S1 and S2 are identical. Since schemes S3-S4 only turn off nodes opportunistically for energy savings, the reception success is the same as S1 and S2.

For each node i with $h_i = |H_i|$ neighbors, the probability that a reception success occurs in a slot is $h_i p_{rx} p_{tx} (1-p_{tx})^{h_i-1}$. The expected number of reception successes in a slot is then $\sum_{i=1}^N h_i p_{rx} p_{tx} (1-p_{tx})^{h_i-1}$. It is easy to show that over all topologies, the maximum expected number of reception successes in a slot occurs for a symmetric topology with $h_1 = \dots = h_N = -1/\ln(1-p_{tx})$ subject to the constraint $h_1, \dots, h_N \in [1, N-1]$. Over all possible state probabilities, the maximum expected number of reception successes occurs with $p_{rx} = \exp(-1/(N-1))$ and $p_{tx} = 1 - \exp(-1/(N-1))$. For large N , the upperbound holds. ■

The expected number of reception successes represents the average data delivery capability of a network, taking advantage of the broadcast nature of the wireless channel. As an example, suppose each node sends heartbeats to its neighbors for neighbor discovery and neighbor list maintenance. In this case, a total of $h_{total} = \sum_{i=1}^N h_i$ successful receptions must take place, which is the sum of all node degrees. Hence, for large N and with optimal state probabilities, the number of slots needed to exchange heartbeats among all neighboring nodes is $O(\bar{h})$, where \bar{h} is the average node degree. As expected, since this traffic pattern consists of only local transmissions, it scales well with the size of the network.

Note that the reception capacity bound is reminiscent of the throughput of the slotted ALOHA system. Indeed, for a node i in the ON-RX state, the neighboring nodes compete for i 's slot in a similar way as a slotted ALOHA system, with p_{tx} dictating the success probability. For large N and $p_{off} = 0$, each node is in the ON-RX state most of the time with p_{tx} diminishing with N and the average number of transmission attempts incident on a node in the ON-RX state being approximately 1. As with a slotted ALOHA system, the total traffic load incident on each node cannot exceed $1/e$ in the best case. For general topologies, the upperbound on the total traffic load at each node is typically smaller than $1/e$. [11] provides a more detailed analysis of the average throughput of multihop networks with regular structure using slotted ALOHA.

Given a topology, we would like to find the optimal state probabilities p_{tx}^* and p_{rx}^* that maximize the expected reception success for schemes S1-S4. Since putting nodes in the OFF state cannot improve the reception success, the optimal state probabilities satisfy $p_{rx}^* = 1 - p_{tx}^*$. This yields

$E[\text{reception success}] = \sum_{i=1}^N h_i p_{tx} (1 - p_{tx})^{h_i}$, which is again topology dependent. For line, Manhattan grid, and triangular mesh networks, one can show that $p_{tx}^* \rightarrow 1/3$, $1/5$, and $1/7$ respectively for large N with reception capacity equal to $0.30N$, $0.33N$ and $0.34N$ respectively.

For uni-cast traffic, the average number of hop-delivery successes is less than the average number of reception successes. Since each node in the ON-TX state selects one of its neighbors with uniform probability to transmit to, the expected number of hop-delivery successes in a slot under message

delivery scheme S1 is $\sum_{i=1}^N p_{rx} p_{tx} (1 - p_{tx})^{h_i - 1} \sum_{j \in H_i} h_j^{-1}$. The ex-

pressions for S2-S6 are difficult to derive since they depend on one-hop and two-hop neighborhood topologies. We show the relationship between average hop-delivery success and state probabilities through simulations in Section IV.

Theorem 4. For a given network topology and an N -tuple of node states $[\psi_1, \dots, \psi_N]$, the hop-delivery success under message delivery schemes S1-S4 and S6 satisfy

$$N_{hop,S1} < N_{hop,S2} = N_{hop,S3} < N_{hop,S4} < N_{hop,S6}$$

Proof: Under S1, a transmitter i randomly chooses one of the neighboring nodes in the set H_i to transmit to. Under S2 and S3, i randomly chooses one of the neighboring nodes in the ON-RX state (according to the original schedule) to transmit to. Let this set be denoted by $H_{i(S2)}$. Under S4, i randomly chooses one of the neighboring nodes in the ON-RX state (according to the modified schedule) to transmit to. Let this set be denoted by $H_{i(S4)}$. Let the set of hop-delivery success node be denoted by $H_{i,hop}$. Since $H_i \supset H_{i(S2)} = H_{i(S3)} \supset H_{i(S4)} \supseteq H_{i,hop}$,

the number of hop-delivery successes increases as the magnitude of the sets decrease. Under S6, the hop-delivery success is further improved upon S4 by additional transmission attempts. ■

Corollary: The Delivery Capacity (DC) under the message delivery schemes satisfy

$$DC_{S1} < DC_{S2} = DC_{S3} < DC_{S4} < DC_{S6}.$$

From the above analysis, it can be seen that exchanging schedule information attains higher number of hop-delivery successes with lower average energy consumption compared to the purely random scheme. By turning off nodes opportunistically, the energy can be further reduced. If two hop schedule information is exchanged, the hop-delivery success can be further improved while the energy is further reduced.

The analysis above does not provide an analytical comparison between schemes S4 and S5 which employ two different collision avoidance schemes. S4 avoids collisions deterministically by choosing a neighboring node that is guaranteed to receive the transmission with success. This may constitute a small set of nodes but each is guaranteed to be successful. On the other hand, S5 avoids collisions probabilistically by scaling back the transmission probability – more receiver nodes have a chance to receive but success is not guaranteed. Through simulations, we show that S4 performs better both in energy and hop-delivery success in all cases simulated.

For a given topology and message delivery scheme S_k , we would like to find $\min_{p_{tx}, p_{rx}} (E_{tx} N_{tx}^{S_k} + E_{rx} N_{rx}^{S_k})$ s.t. average hop-delivery success = γ , where γ is the desired average hop-delivery success. By varying γ , an optimal average energy and average hop-delivery success tradeoff curve can be obtained. We explore this tradeoff curve through simulation.

IV. SIMULATION RESULTS

To better illustrate the tradeoffs between energy and hop-delivery success as well as the optimal state probabilities that generates the energy and hop-delivery capacity tradeoff curve, we perform simulations on the following topologies with 100 nodes: 1. A regular line network. 2. A regular square grid network. 3. A randomly generated network with nodes placed uniformly and randomly in a unit-area square. Two nodes are connected if their distance is below a threshold $r = 0.2$.

For each topology and for each message delivery scheme S1-S6, the average hop-delivery success and energy is computed for a range of state probabilities, where the average is taken over 1000 slots. For the purpose of simulation, we took $E_{tx} = 1.5E_{rx}$ since the amount of energy expended in transmission typically exceeds the amount of energy expended in reception. All of the subsequent discussions hold for higher energy ratios.

Fig. 1 shows the optimal average energy and average hop-delivery success tradeoff curves for the three topologies, where the average energy is normalized by the maximum energy attained by all schemes. These tradeoff curves are achieved by optimally tuning state probabilities (p_{tx}, p_{rx}) .

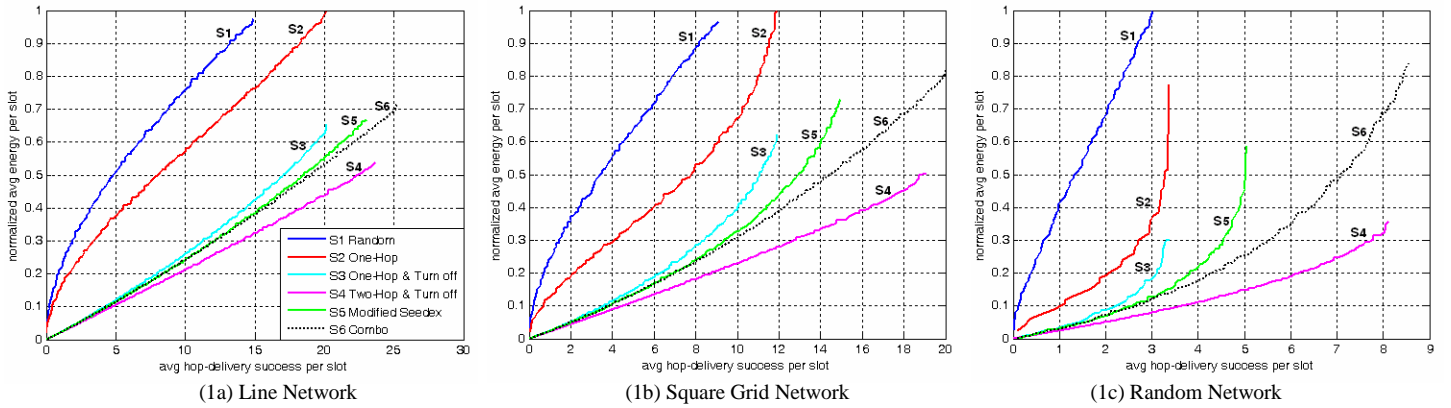


Fig. 1 Average Energy vs. Average Hop-Delivery Success (the delivery capacity under each scheme = maximum average hop-delivery success for each scheme)

Clearly, S1 and S2 have much worse energy vs. hop-delivery success tradeoffs than the other four schemes. In particular, S1 achieves the lowest delivery capacity, followed by S2 and S3 (which achieve equal delivery capacity as expected), S5, and S4, while S6 achieves the highest delivery capacity. One would expect that the delivery capacity can be improved by exchanging schedule information with k -hop neighbors in the network, where $k > 2$; however, the gain diminishes with k . For very small average hop-delivery successes, S3-S6 have very similar energy consumptions. In this case, S3 with one-hop schedule exchange is sufficient. However as average hop-delivery successes increase, the average energy consumption differences become more pronounced. The amount of energy savings of S4 over the other schemes also become more pronounced as the average node degree increases.

For randomly generated networks with a constant transmission range threshold r , the results are fairly independent of the actual topology. As r increases, the average node degree increases, while the delivery capacity of the network decreases. This should be contrasted with the reception capacity which tends to increase with increase in average node degree as shown in Section III. Fig. 2 shows the average delivery capacity decreases exponentially with increase in r for a randomly generated network under scheme S4. The exponential decay holds for all other schemes as well. This should not be taken to imply that the transmission range of a network should be as small as possible. As alluded to earlier, the end-to-end throughput of a network depends on the spatial reuse and the amount of pass-through traffic generated by traffic routing. Although it is desirable to achieve high spatial reuse for a given routing scheme as we have discussed in this paper, the overall problem of improving end-to-end throughput and energy consumption, which involves a joint optimization of the message delivery scheme and routing, has not been addressed in this paper. This joint optimization is topology and traffic dependent and is subject to future research.

As mentioned, the optimal tradeoff curves of Fig. 1 are obtained by tuning state probabilities. These curves are difficult to characterize analytically but are fairly easy to compute numerically. Fig. 3 shows contour plots of average energy and hop-delivery successes superimposed on one another for each of the six message delivery schemes for a randomly generated network. Each average hop-delivery success contour line traces the state probability pairs (p_{tx}, p_{rx}) that obtain the same

average hop-delivery success. By following the average hop-delivery success contours, one can find the state probabilities (p_{tx}, p_{rx}) at which the lowest energy contour line crosses.

For all the schemes, the optimal tradeoff curve is typically obtained by having a higher p_{rx} than p_{tx} . This is intuitive since $E_{tx} > E_{rx}$ and that having more transmitters than receivers can only cause unnecessary collisions. Ideally, the average energy consumption and average hop-delivery success contour lines should have the same shape (ie. energy consumption is proportional to hop-delivery success). Note that of all the schemes, S4 best approximates overlap in the contours.

V. DISCUSSION

We note that, with an additional feature of turning receivers off immediately upon detection of unintentional traffic (assuming each time slot is much longer than the time it takes to decode the header of a packet), our scheme S4 can achieve the near-optimal energy attainable by perfect transmission scheduling for all average hop-delivery successes less than the hop-delivery capacity of S4. This is a direct consequence of Theorem 2 which states that S4 achieves the minimum number of transmissions needed for a particular hop-delivery success. With the additional feature of turning extraneous receivers off and assuming the amount of energy needed to read the header of a packet is negligible, the following holds:

$$N_{hop-delivery} = N_{tx}^{S4}(\psi') = N_{rx}^{S4}(\psi'), \forall \psi' \in \Omega^{S4} \mid N_{hop-delivery} \text{ is achieved.}$$

This is a significant result of tremendous practical value since S4 achieves the minimum energy by the simple random duty-cycling protocol with very small amount of local state information exchange compared to optimal scheduling which requires global state information exchange and coordination

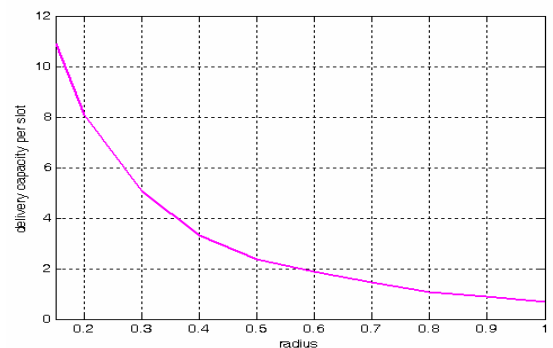


Fig. 2 Average Hop-Delivery vs. Transmission Radius under S4

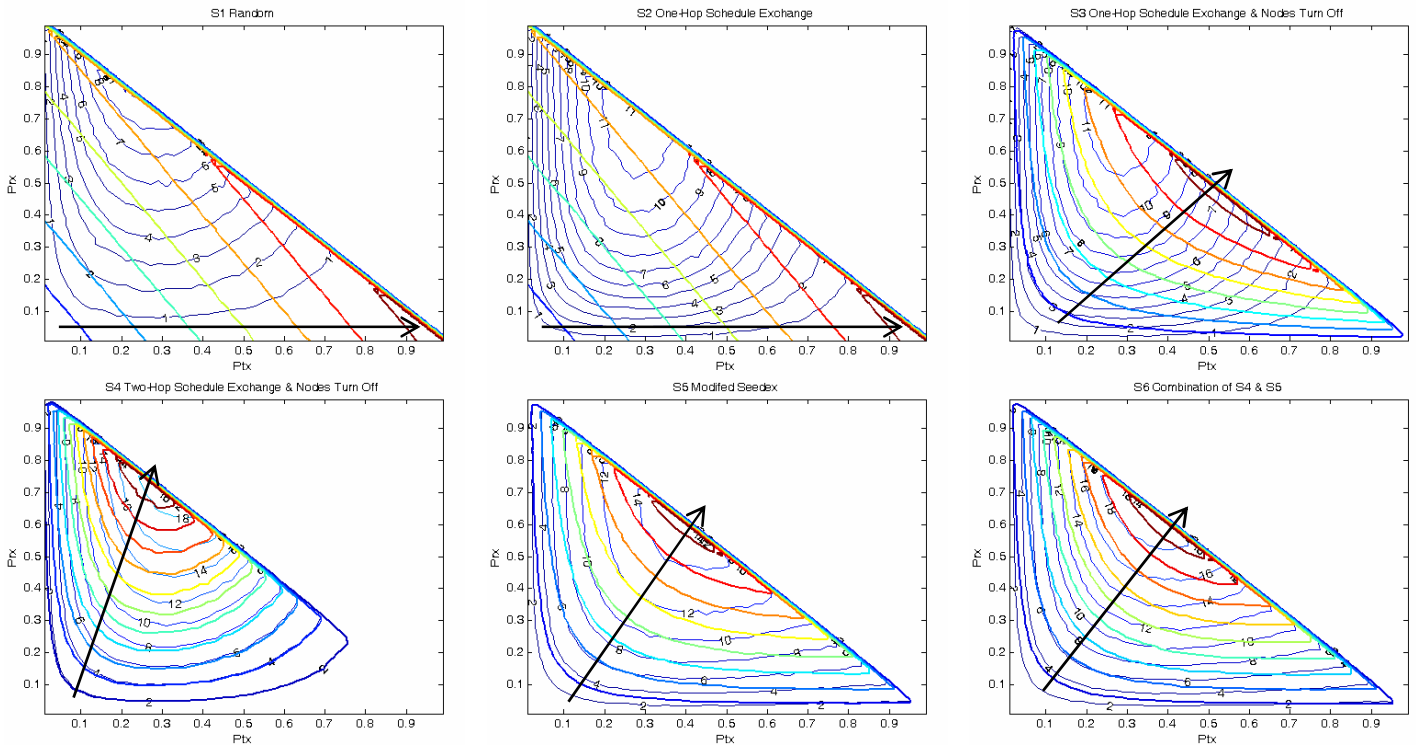


Fig. 3 Average Energy and Hop-Delivery Success Contour Plots for a Square Grid Network (thin blue lines - contours of the average hop-delivery success, with average hop-delivery successes labeled on each contour line thicker colored lines - contours of the average energy, increasing from blue to burgundy in the direction of the arrows)

in every slot. Hence, S4 with the additional receiver turn off feature is an ideal message delivery scheme for WSNs with low traffic loads (below the delivery capacity of S4). This additional feature is not conventionally supported by commodity transceiver hardware.

VI. CONCLUSION

We have analyzed six random message delivery schemes in terms of average energy consumption and traffic delivery capabilities for uni-cast and broadcast traffic. It has been shown that our proposed scheme S4 incorporating two-hop schedule exchange and strategies to opportunistically turn off nodes outperforms all other schemes in terms of average energy consumption. In addition, another proposed scheme S6 that combines opportunistic node turn off and transmission backoff achieves the highest delivery capacity. Overall, S4 and S6 outperform schemes proposed in [4] and [5] both in energy and traffic delivery capabilities. In particular, S4 with an additional feature of turning receivers off upon detection of unintentional traffic achieves near-optimal energy consumption approaching that achievable by perfect transmission scheduling. Its simplicity in implementation combined with optimal energy performance makes it an ideal message delivery scheme for WSNs with low traffic loads.

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