Abstract—The increasing popularity of smartphones and other similar multi-modal wireless devices has created an opportunity for the realization of large-scale hybrid (or heterogeneous) networks. Typically, modern mobile devices are likely to support a short range communication interface (e.g., IEEE 802.11/WiFi) and/or a longer range communication interface (e.g., cellular data link wireless technology). Multi-hop wireless networking over WiFi can help to extend the range of cellular networks in low SINR regions as well as to alleviate network congestion. Conversely, equipping a few nodes in a mobile ad hoc network (MANET) with cellular radios can help to heal wireless network partitions and, thus, to improve the overall network connectivity. One can envision large scale group communication (or multicast) applications including real-time video conferencing (e.g., iPhone FaceTime), P2P video and file sharing, and “voice call groups” in disaster relief and military hybrid networks. In this paper, the problem of resource-efficient multicast in hybrid wireless networks which include both point-to-point (cellular) and broadcast (MANET) links is considered. The underlying optimization problem is a hybrid of two well-known NP-hard graph optimization problems—the Minimum Steiner Tree problem (for point-to-point links) and the Minimum Steiner Connected Dominating Set problem (for broadcast links). We consider both edge- and node-weighted versions of this problem and use distinctly different methodologies to formulate two algorithms with guaranteed approximation factors. We further demonstrate by means of simulation modeling of standard deployment scenarios that while one algorithm outperforms the other in terms of the tree cost, the latter outperforms the former in terms of complexity and other practical considerations. Finally, using algorithmic ideas from percolation theory, we demonstrate the trade-off between network connectivity and multicast cost, when hybrid capability is added incrementally to the nodes of a MANET-only deployment.

I. INTRODUCTION

The increasing popularity of smartphones and other similar multi-modal wireless devices have created an opportunity for the realization of large-scale hybrid (or heterogeneous) networks. Typically, modern mobile devices are likely to support IEEE 802.11/WiFi, which have a short communication range, and/or cellular data link wireless technologies, which have a longer communication range. Figure 1 gives a schematic of a nominal hybrid network architecture.

Multi-hop wireless networking over WiFi can help to extend the range of cellular networks in low SINR (Signal to Interference plus Noise Ratio) regions as well as to alleviate network congestion [15]. Conversely, equipping a small fraction of nodes in a mobile ad hoc network (MANET) with cellular radios can help heal wireless network partitions and, thus, improve the overall network connectivity. This proves useful in the military and sensor networks/DTN contexts where islands of connectivity are formed no matter how hard one tries to make the MANET connected, e.g., CERDEC MACE program [4]. One can also envision large scale group communication (or multicast) applications including real-time video conferencing (e.g., iPhone FaceTime), P2P video and file sharing, and “voice call groups” in disaster relief and military hybrid networks.

In this paper, we consider the problem of resource-efficient multicast in hybrid wireless networks which include both point-to-point (cellular) and broadcast (WiFi) links. The underlying optimization problem is a hybrid of two well-known NP-hard graph optimization problems—the Minimum Steiner Tree problem (for point-to-point links) and the Minimum Steiner Connected Dominating Set problem (for broadcast links). We consider both edge- and node-weighted versions of this problem and use two distinctly different methodologies to give two algorithms with guaranteed approximation factors.

Network multicast is an important topic to study in such hybrid networks. Smartphone users like sharing real-time video or want to perform videoconferencing with friends, which forms a multicast group [13]. Even without P2P technologies enabled on the smartphone, one can imagine a simple use case of a hybrid network consisting of 4G cellular (point-to-point) + 802.11 WiFi (last hop broadcast) [9]. Other examples of multicast include “voice call groups” in military and disaster relief networks, and video sharing in P2P wireless is a popular problem [7]. Finally, “link state” routing algorithms may need efficient network-wide broadcast of link state updates.

We believe that the following categories of hybrid networks stand to benefit from efficient multicasting:

**Tactical Networks.** Mission-critical operations for field and disaster relief missions often rely on a combination of wireless...
ad hoc networks (which can usually be swiftly deployed) and on-site cellular networks (these may be limited in availability). An example of such a network is described in [4].

**Mobile Content Distribution Networks.** The proliferation of mobile gadgets facilitates peer-to-peer communication among users, which may be more economical than, but not as reliable as the wireless data connection through base stations.

**Vehicular Networks.** The advance of vehicular technology enables dedicated short-range communications (DSR) equipped in the emerging new vehicles to receive traffic alerts. It is also possible to employ DSR for distributing wide-area traffic information among vehicles, in addition to connections through road-side kiosks.

### A. Technology types in a hybrid network

While the set of various available technology types of the nodes in the hybrid network can be very general, in wireless hybrid networks, nodes could typically possess one or more of the following distinct types of wireless technologies:

1) **Cellular** wireless via base stations (macrocells, femtocells, etc.)—in this case, any two smartphones equipped with the cellular SIM card can potentially communicate via a series of base stations (BS). The quality of this “cellular” link between two smartphones will depend on how far they are located with respect to their closest BSs, respectively. Thus, the set of cellular-capable nodes form a weighted *clique*. This can be generalized to a non-clique where broadcast technologies in cellular networks [6] can be exploited as described in Section VII; or even a disconnected graph.

2) **Wireless** point-to-point mesh—in this case, any two nodes equipped with the specific interface hardware can potentially communicate directly if they are within the transmission range of each other. However, a node does not communicate with multiple nodes in range using a single broadcast transmission. The set of mesh nodes form a *random geometric graph*.

3) **MANET** wireless broadcast—this is similar to the wireless mesh setting with the exception that a node can reach multiple nodes in its transmission range using a single broadcast transmission. The set of MANET nodes form a *random geometric directed hypergraph*.

### B. Our Contributions

In this paper, we focus on hybrid networks composed of cellular and MANET technologies since they represent point-to-point wireless and wireless broadcast advantage. We explore theoretical/algorithmic aspects of the hybrid network multicast problem using standard deployment modelling scenarios. Specifically, we propose and study

1) A directed hypergraph model for edge-weighted hybrid networks and an approximation algorithm for a minimum cost multicast based on the approximation algorithm for directed Steiner problem by Charikar et al. [3].

2) A $3 \log_2 |M|$ heuristic approximation algorithm for the unweighted hybrid network multicast problem, which tries to minimize the total number of transmissions irrespective of the wireless network costs. We derive a pseudo-polynomial approximation ratio for the heuristic algorithm when the unweighted algorithm is applied to a weighted version of the problem.

3) Systematic evaluation of tradeoffs between the performance of the aforementioned algorithms in realistic wireless hybrid network scenarios.

We also use an algorithm from percolation theory [11], [10] to numerically evaluate the trade-off space between multicast cost and network connectivity of a hybrid network. We show that incremental addition of hybrid capability to nodes of a MANET-only network (i) progressively reduces the network’s site-percolation threshold (long-range connectivity appears with a smaller node density), and (ii) reduces the multicast cost, both when the original MANET-only network is in the sub-critical and super-critical connectivity regimes.

To the best of our knowledge, we are the first to study multicast in hybrid networks that combine wireless broadcast capability and point-to-point wireless links.

### C. Related Work

Hybrid wireless networks are a well researched topic. Researchers have studied various aspects of it, including capacity scaling laws [8], connectivity scaling [17], application oriented routing [14], etc. However, the problem of multicast in such networks has not received adequate attention.

Our optimization problem is a hybrid of Steiner Tree problem for point-to-point links and Steiner Connected Dominating Set (CDS) problem for broadcast links, both of which are NP-hard. Efficient algorithms for solving Steiner CDS problem, which captures the wireless version of multicast problem, was first studied by Guha and Khuller [5]. In their paper, they presented a polynomial-time $O(\log n)$-approximation algorithm for the *unweighted* Steiner CDS problem, and showed a setting in which that there exists no polynomial-time algorithm of any polynomial approximation factor for the weighted Steiner CDS problem. Recently, Wu, Xu, and Chen [16] presented a slightly improved algorithm for Steiner CDS problem. In planar or unit-disk graphs, CDS problems are more tractable, approximable within a constant factor. However, these graphs cannot realistically capture hybrid networks, especially because the wireline part may not be restricted by the geometry.

Recently, Panigrahi [12] presented a class of problems called activation network design problem, in which the cost of a link depends on the states of its end-nodes through an activation function. The activation network problem does not precisely capture the hybrid network multicast problem, although CDS can be mapped to an activation network problem with a constant approximation ratio.

Recently, we have investigated the multicast problem under multi-domain and hierarchical constraints [2].

### II. PRELIMINARIES AND PROBLEM FORMULATION

We first formally describe the traditional multicast problem before explaining the hybrid network multicast problem.
Given a graph $G = (V, E)$, there is a non-empty set of terminals (or sinks) $M \subseteq V$, which are required to participate in a multicast communication. The case $|M| = 2$ corresponds to unicast communication, whereas $M = V$ corresponds to network-wide broadcast. For a given graph $G$ and a subset of its nodes $M$ a Steiner tree is a tree in $G$ that spans all vertices of $M$. By writing $T = (V(T), E(T))$ we typically mean a subgraph of $G$ with $V(T) \subseteq V$ and $E(T) \subseteq E$.

### A. Basic Multicast Network Problems

**Edge-Weighted Steiner Tree:** Assume that each edge $e \in E$ has an associated cost $\gamma(e)$. The elements of $V \setminus M$ are called Steiner nodes—these may participate in multicast communication if their involvement can enable connectivity between terminals in $M$.

**Definition 1.** Edge-weighted Steiner tree problem consists in minimizing $\sum_{e \in E(T)} \gamma(e)$ over $T$ subject to: (i) $T$ is a connected subgraph of $G$, and (ii) each pair of nodes $u, v \in M$ are connected via $T$.

**Node-Weighted Steiner Tree:** Assume that each node $v \in V$ has an associated cost $c(v)$. Using previously defined notation, we define the following node-weighted Steiner tree problem.

**Definition 2.** Node-weighted Steiner tree problem consists in minimizing $\sum_{v \in V(T)} c(v)$ over $T$ subject to: (i) $T$ is a connected subgraph of $G$, and (ii) each pair of nodes $u, v \in M$ are connected via $T$.

**Connected Dominating Set:** A set $C \subseteq V$ is called dominating set of $B$ in $G$, if for every $v \in B \setminus C$ there exists $u \in C$, such that $(u, v) \in E$.

**Definition 3.** Node-weighted connected dominating set problem consists in minimizing $\min_{C} \sum_{v \in C} c(v)$ over $C$ subject to: (i) $C$ is a dominating set of $V$, and (ii) if $u, v \in C$, there is a path via $u, v_1, ..., v_r, v$, such that $v_1, ..., v_r \in C$.

**Steiner Connected Dominating Set:**

**Definition 4.** Node-weighted Steiner connected dominating set problem consists in minimizing $\sum_{v \in C} c(v)$ over $C$ subject to (i) $C$ is a dominating set of $M$, and (ii) if $u, v \in C$, there is a path via $u, v_1, ..., v_r, v$, such that $v_1, ..., v_r \in C$.

### B. Generalized Hybrid Multicast Network Problem

We consider a hybrid network represented by a multi-graph $G = (V, E)$ characterized as follows. The vertex set $V$ is split into three partitions:

1. $V_{\text{manet}}$ (for wireless MANET nodes—type $M$ for short),
2. $V_{\text{cell}}$ (for wireless cellular nodes—type $C$ for short),
3. $V_{\text{hyb}}$ (for hybrid nodes with both MANET and cellular interfaces—type $H$ for short)

Each link $(u, v) \in E$ represents a communication link between a pair of nodes, where $E$ is split into the following two partitions:

1. $E_{\text{manet}}$ (for wireless MANET links),
2. $E_{\text{cell}}$ (for cellular links)

where

- $(u, v) \in E_{\text{manet}}$ implies $u, v \in V_{\text{manet}} \cup V_{\text{hyb}}$,
- $(u, v) \in E_{\text{cell}}$ implies $u, v \in V_{\text{cell}} \cup V_{\text{hyb}}$.

Denote by $H = (V(H) \subseteq V, E(H) \subseteq E)$ a subgraph of $G$ and consider a set of terminals $M \subseteq V$. Denote the set of leaf nodes in a subgraph $H$ by $\text{Leaf}(H)$. Also, let us define the set of nodes connected by wireless broadcast links in $H$ by

$$V_{\text{manet}}(H) \triangleq \{ v \in (V_{\text{manet}} \cup V_{\text{hyb}}) \cap V(H) | \exists (u, v) \in E_{\text{manet}} \cap E(H) \}$$

Let $c(v)$ and $\gamma(e)$ denote the costs of a node $v$ and a link $e$, respectively. In this paper we consider scenarios with additive costs, i.e., where the cost of a multicast subgraph is the sum of the costs of the nodes and edges contained in that subgraph. An important example of such a family of additive costs is “energy”. Other non-additive multicast costs such as “latency” are left for future research.

Given a subgraph $H$, we define the cost as a sum of costs due to all MANET broadcasts (which are captured in terms of non-leaf node costs since a MANET node has to pay the cost for all its adjacent MANET links only once) or cellular point-to-point links. Since the leaf nodes do not incur any cost in transmitting a message, their costs are not counted.

$$\text{Cost}(H) \triangleq \sum_{v \in V(H) \setminus \text{Leaf}(H) \cap V_{\text{manet}}(H)} c(v) + \sum_{e \in E(H) \cap E_{\text{cell}}} \gamma(e)$$

**Definition 5.** Generalized hybrid multicast network problem consists in minimizing $\text{Cost}(H)$, as above in (1), with respect to $H$ subject to the following: (i) $H$ is a connected subgraph of $G$, and (ii) each pair of nodes $u, v \in M$ are connected via $H$.

### III. HYBRID NETWORK MODELS AND APPROACH

Both edge and node based costs are possible in the problem formulation presented in Section II. Owing to the presence of both broadcast and point-to-point links in a hybrid network, care must be taken to model the costs appropriately. Typically, there are a few different types of costs that may be encountered in hybrid networks.

1. When a MANET or a hybrid node $v$ performs a wireless broadcast, all the MANET links adjacent to this node are simultaneously activated, hence the cost charged to a broadcast can be transferred from the links to the node instead.
2. When a cellular or a hybrid node $v$ performs a transmission over a cellular link to another cellular or hybrid node, each cellular link incident on $v$ constitutes a separate cost.
3. Additional costs on the nodes during the multicasting operation.

We propose two somewhat related but distinct graph constructions for addressing both the cases when only links are charged a cost and when both links and nodes are charged costs. The high-level procedure is outlined in Figure 2.
A. Constructing Edge-weighted Graphs from Directed Hypergraphs

Consider a directed hypergraph \( H = (V, E) \) where \( V \) is a set of vertices and \( E \) is a set of hyperedges. A hyperedge \( e \in E \) is an ordered pair \((V_t, V_H)\) of non-empty, disjoint subsets of \( V \) referred to as the tail vertices, \( V_t(e) \), and the head vertices, \( V_H(e) \), of the hyperedge \( e \), respectively. For each hyperedge \( e \) we associate a cost \( \gamma(e) \in \mathbb{R} \). Define the incidence matrix, \( I_H \), of size \(|V| \times |E|\) for hypergraph \( H \) by

\[
(I_H)_{v,e} = \begin{cases} 
+1 & \text{if } v \in V_H(e), \\
-1 & \text{if } v \in V_t(e), \\
0 & \text{otherwise}.
\end{cases}
\]  

(2)

Given the hypergraph \( H = (V, E) \) define the associated bipartite graph \( G' = (V', E') \) where \( V' \) is partitioned into \( V' = V'_L \cup V'_R \) with \( V'_L = V \), \( V'_R = E \) and \( E' = \{(v, e) \mid v \in V \land e \in V_t(e)\} \cup \{(e, v) \mid e \in E \land v \in V_H(e)\} \). We now augment the bipartite graph \( G' \) as follows to form a new (non-bipartite) graph \( G'' \). For each node, \( e \), of \( G' \) corresponding to a hyperedge in \( V'_R \) replace that node by two nodes \( e^- \) and \( e^+ \) together with a directed edge \((e^-, e^+)\) and attach the cost to that edge given by the cost \( \gamma(e) \) of the hyperedge \( e \) in the original hypergraph \( H \). We attribute zero costs to all remaining edges in \( G'' \).

Example: \( H = (V, E) \) where \( V = \{v_1, v_2, v_3, v_4, v_5\} \) and \( E = \{e_1, e_2, e_3\} \) where \( e_1 = \{(v_1), (v_2, v_3)\} \), \( e_2 = \{(v_3), (v_1, v_2, v_4)\} \) and \( e_3 = \{(v_3), (v_2)\} \). The incidence matrix and the graphs \( G' \) and \( G'' \) are as shown in Figure 3.

B. Directed Hypergraph Models for Hybrid Networks

A natural way of modeling hybrid networks with edge costs is using a directed hypergraph with weights on its hyperedges.

A MANET/cellular hybrid wireless network consists of edges that belong to the following categories:

1) Cellular point-to-point edges: In our model, any (C,C) or (C,H) node pairs may have connectivity since the cellular base station network is assumed to be connected via wireline networks. The communication cost of a cellular link is a function of its quality. This is typically an edge cost which depends on the position of the endpoints of the link with respect to their nearest base stations. In this paper, we use the following cost function for a cellular edge denoted by \((u,v)\):

\[
c_{uv} = \beta_u d(u, BS_u)^{\alpha_c} + \beta_v d(v, BS_v)^{\alpha_c},
\]

(3)

for some node-specific constants \( \beta_u, \beta_v \) and a path loss exponent \( \alpha_c \in [2,5] \). Here \( d(u, BS_u) \) is the physical distance between \( u \) and its nearest base station node \( BS_u \). Ideally, the choice of \( \beta_u \)'s will depend on the actual real cellular deployment data but here we assume \( \beta_u = 1 \) for all cellular/hybrid nodes \( u \). For each occurrence of a cellular edge we construct a directed hyperedge \( e \) and assign it a cost \( \gamma(e) = c_{uv} \).

2) MANET broadcast edges: Each MANET broadcast can be performed by a MANET (M) node or a hybrid (H) gateway node, and can simultaneously reach all its M neighbors within a fixed radio transmission range \( R \). If a MANET node \( u \)'s broadcast reaches one or more recipients, say \( v_1, v_2, \ldots, v_k \), we construct a directed hyperedge \( e = \{u \rightarrow v_1, v_2, \ldots, v_k\} \) and assign it a cost \( \gamma(e) = R_{m}^\alpha \), where \( \alpha_m \) is the pathless exponent for MANETs. Note that typically \( \alpha_m \geq \alpha_c \) since MANET nodes are located closer to the ground.

3) Example: The above construction is illustrated by an example in Figure 4. Cellular links are indicated by dotted...
the lines, whereas MANET links are indicated by dashed lines. The network in Figure 4 can be modeled by the following directed hypergraph on 10 nodes \( \{a, b, c, d, e, f, g, h, i, j\} \) and the following 21 hyperedges:

1: \( a \to b, c \)  
2: \( a \to f \)  
3: \( a \to g \)  
4: \( a \to h \)  
5: \( b \to a, c, d, e \)  
6: \( c \to a, b, d, e \)  
7: \( d \to b, c, e, f, g, i \)  
8: \( e \to b, c, d, f, g, j \)  
9: \( f \to d, e, i \)  
10: \( f \to a \)  
11: \( f \to g \)  
12: \( f \to h \)  
13: \( g \to d, e, j \)  
14: \( g \to a \)  
15: \( g \to f \)  
16: \( g \to h \)  
17: \( h \to a \)  
18: \( h \to f \)  
19: \( h \to g \)  
20: \( i \to d, f \)  
21: \( j \to e, g \)

This can then be converted to a purely edge-weighted graph by following the construction in Section III-A. It is easy to see that the following two directed sub-hypergraphs are reasonable solutions connecting the root \( a \) with terminals \( f, g, h \):

1) \( T_1 = \{a \to f; f \to d, e, i; e \to c, d, g, j; f \to h\} \)
2) \( T_2 = \{a \to b, c; b \to d, e; d \to b, c, e, f, g, i; e \to c, d, g, j; f \to h\} \)

If a unicast costs \( u \) and a broadcast costs \( b \) units of energy (ignoring location of mobiles and hence signal strength for the moment), \( \text{Cost}(T_1) = 2u + 2b \), whereas \( \text{Cost}(T_2) = u + 4b \). Hence either \( T_1 \) or \( T_2 \) may be optimal depending on the relative values of \( u \) and \( b \).

In Section IV-A, we use the aforementioned construction and give an approximation algorithm for computing the optimal Steiner hypergraph for solving the hybrid network multicast problem.

C. Multigraph Models for Hybrid networks

We consider now the general version of the hybrid network multicast problem where there are not only costs on the cellular point-to-point links (C-C, C-H, H-H), but also potentially on the nodes themselves. To tackle this scenario, we propose to model the hybrid network as a multigraph. Essentially, for all links \( (u, v) \), if both \( u, v \) are of type \( H \), we construct two edges between them in the multigraph. This augmentation construction is illustrated in Figure 5.

In Section IV-B we give another approximation algorithm which works with the multigraph input instead of the hypergraph input. The advantage of the multigraph input is that it can be converted to a simple undirected node weighted graph with zero edge costs (cellular edges are converted to dummy nodes and edge weights are transferred to them). This construction allows us to combine techniques from Connected Dominating Sets and Node Weighted Steiner Trees for solving the hybrid network multicast problem.

IV. ALGORITHMS FOR HYBRID NETWORK MULTICAST

A. Hypergraph case

For this case, we first use the technique of transforming a given edge-weighted hybrid wireless multigraph into a directed hypergraph (see Section III). We then use a standard method to transform this directed hypergraph into a bipartite graph with costs on nodes that correspond to hyperedges in the original. Finally, we further augment the obtained bipartite graph into a directed graph by representing each non-negative cost node by a directed edge and transfer the node costs onto newly created dummy nodes and edge weights are transferred to them). This construction allows us to combine techniques from Connected Dominating Sets and Node Weighted Steiner Trees for solving the hybrid network multicast problem.

THEOREM 1. Algorithm HYB-DIRECTED-MULTICAST yields a \( O(|M|^c) \) approximation algorithm for \( c > 0 \).

Proof: We need to show that the \( G'' \), as described in Section III-A, although typically larger in size compared to the original network, still has \( O(poly(n)) \) nodes.

Note that in the worst case (a complete graph formed by \( n \) hybrid nodes), since a node can appear at the tail of at most
Algorithm 1 DIRECTED STEINERTREE $A(G, r, X, i, k)$

Require: An edge-weighted directed graph $G$, a node $r \in G$; a subset of nodes $X \subseteq G$; $i$ and $k$ are positive integers.

Ensure: A tree $T$ rooted at $r$ that reaches at least $k$ terminals in $X$.

1: if there do not exist $k$ terminals in $X$ reachable from $r$ then return $\emptyset$
2: if $i == 1$ then
3: find $k$ terminals closest to $r$ and output the shortest (lowest cost) paths tree rooted at $r$;
4: $T \leftarrow \emptyset$;
5: while $k > 0$ do
6: $T_{BEST} \leftarrow 0$; $d(T_{BEST}) \leftarrow \infty$;
7: for each vertex $v \in G$ do
8: for $k' = 1$ to $k$ do
9: Find lowest cost path $P_{r \rightarrow v}$ from $r$ to $v$ in $G$;
10: $T' \leftarrow A(G, v, X, i - 1, k') \cup P_{r \rightarrow v}$;
11: if $d(T_{BEST}) > d(T')$ then
12: $T_{BEST} \leftarrow T'$; $d(T_{BEST}) \leftarrow d(T')$;
13: $T \leftarrow T \cup T_{BEST}$; $k \leftarrow k - |X \cap V(T_{BEST})|$;
14: return $T$

Algorithm 2 HYB-DIRECTED-MULTICAST ($\mathcal{H}, \mathcal{M}, c(\cdot)$)

1: Generate a bipartite graph $G'$ from $\mathcal{H}$ following the steps in Section III-A. The nodes to the right side of $G'$ correspond to hyperedges, and hence have costs associated with them.
2: Generate an edge-weighted non-bipartite graph $G''$ from $G'$ following the steps in Section III-A. All nodes in $G''$ have zero cost.
3: $\mathcal{R} = \text{ROOT} (\mathcal{M}) \triangleright$ One of the terminals must be a root.
4: $\mathcal{H}_T \leftarrow \text{DIRECTED STEINERTREE} (G'', \mathcal{R}, \mathcal{M})$.
5: return $\mathcal{H}_T \triangleright$ Approx. optimal hybrid multicast tree.

Theorem 2. Consider unweighted nodes. Algorithm HYB-MULTICAST (Algorithm 3) is a polynomial-time $3 \log(|\mathcal{M}|)$ approximation algorithm.

Proof: First, we note that for a minimum-cost Steiner connected dominating set (StCDS) and a minimum-cost Steiner tree (ST) for a graph, we have:

\[
\text{Cost}(\text{StCDS}) \geq \text{Cost}(\text{ST}),
\]

since the non-leaf terminals still count in StCDS (see example of the weighted case in Figure 1 of [2]).

Let $\text{Cost}(\text{Opt})$ be the cost of optimal hybrid multicast network (partly StCDS and partly Steiner tree). Consider the augmented terminal set $A \cup (M \setminus B_M)$ consisting of terminals of type C and nodes in the neighborhoods of terminal nodes of type M/H; then Opt will also connect $A \cup (M \setminus B_M)$ ($A$ is defined in Algorithm 3). However, when comparing the minimum-cost Steiner tree $T$ that connects this set, we have:

\[
\text{Cost}(\text{Opt}) \geq \text{Cost}(T).
\]

Let $S^*$ be the optimal dominating set for $M \cap (V_{\text{wh}} \cup V_{\text{gray}})$, i.e., only the terminal nodes with MANET broadcasting capabilities. Thus, we obtain ($K$ is defined in Algorithm 3):

\[
\log(|\mathcal{M}|)|S^*| \geq |K|.
\]

Also, by node-weighted Steiner tree approximation algorithm (e.g., Klein-Ravi):

\[
\log(|\mathcal{M}|)\text{Cost}(T) \geq \text{Cost}(\mathcal{H}).
\]
Applying the inequalities, we obtain:
\[
3 \log(M) \text{Cost}(\text{Opt}) \\
\geq \log(M) \text{Cost}(\text{Opt}) + |A| + \log(M)|S^*| \\
\geq \log(M) \text{Cost}(T) + |A| + |K| \\
\geq \text{Cost}(H) + |A| + |K| \\
\geq \text{Cost}(\text{HYB-MULTICAST}).
\]

**Corollary 3.** Consider weighted nodes, such that the minimum weight is 1 and maximum weight is \( w_{\text{max}} \). Algorithm \text{HYB-MULTICAST} is a polynomial-time \( 3w_{\text{max}} \log(|M|) \) approximation algorithm.

**V. PERFORMANCE EVALUATION**

In this section we study the relative performance trade-offs of the two algorithms proposed in this paper by means of numerical simulations. We generate several different hybrid network deployment scenarios, the parameters for which are numerical simulations. We generate several different hybrid network deployment scenarios, the parameters for which are given in Table I.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>PPP density parameter</td>
<td>0.33</td>
</tr>
<tr>
<td>( L \times L )</td>
<td>Area of deployment</td>
<td>10 \times 10</td>
</tr>
<tr>
<td>( n )</td>
<td>Total number of nodes</td>
<td>Poisson (mean ( \lambda L^2 ))</td>
</tr>
<tr>
<td>( P_M : P_C : P_H )</td>
<td>Relative deployment ratios</td>
<td>7 : 1 : 2</td>
</tr>
<tr>
<td>(</td>
<td>M</td>
<td>)</td>
</tr>
<tr>
<td>( h )</td>
<td>Cell diameter of honeycomb</td>
<td>2</td>
</tr>
<tr>
<td>( R )</td>
<td>MANET radius of connectivity</td>
<td>1.9</td>
</tr>
<tr>
<td>( \alpha_m )</td>
<td>MANET path loss coefficient</td>
<td>4</td>
</tr>
<tr>
<td>( \alpha_c )</td>
<td>Cellular path loss coefficient</td>
<td>3</td>
</tr>
</tbody>
</table>

The following set of steps outlines the simulation procedure:
1) Lay down a honeycomb lattice structure for simulating a cellular base-station network.
2) Generate node locations according to a 2D Poisson Point Process with the density \( \lambda \). This yields a total of \( n \) nodes where \( n \) is Poisson distributed with mean \( \lambda L^2 \).
3) According to the \( P_M : P_C : P_H \) ratio, assign randomly MANET, cellular, or hybrid capability to each node.
4) Create \( M–M \), \( M–H \), \( H–M \), and \( H–H \) broadcast links if the corresponding nodes are located within the corresponding MANET transmission radius \( R \); assign cost \( R^{\alpha_m} \) to these transmissions. Note that these costs will be assigned to hyperedges for Algorithm 2 but to nodes if Algorithm 3 is to be used.
5) Create \( C–C \), \( C–H \), \( C–H \), \( H–H \) point-to-point links for all such pairs; assign costs to these links following the procedure from Section III-B.
6) Choose set \( M \) of terminals randomly in the largest connected component of the resultant hypergraph or multigraph.
7) Execute Algorithms 2 and 3 and compare performance.

The following figures show the performance comparison of the algorithms.

Fig. 6. Sample multicast trees yielded by Algorithms 2 and 3. The terminal nodes are 8, 42, 4, 7, 12.

Fig. 7. Performance comparison between the total cost of the multicast tree for Algorithms 2 (red dots) and 3 (black dots and bars for the mean and standard errors with 25 independent runs).

Fig. 8. This figure shows how the total cost of the hybrid multicast tree grows linearly with the number of terminals ranging between 4 and 21 nodes.

Figure 6 shows a sample output given by both algorithms (overlaid on the same geographic area). In this case Algorithm 2 took a cheaper cellular link instead of Algorithm 3.
In this section we analyze the trade-space between connectivity and multicast cost over a 2D spatial hybrid network with \( n \) nodes deployed with a Poisson rate \( \lambda \) over an \( L^2 \)-area square region (i.e., \( n \sim \lambda L^2 \)) and a MANET-connectivity range \( R \).

We study how the overall connectivity of such a MANET-only network improves as nodes are progressively equipped with cellular technology. This analysis can inform the choice of parameters for practical hybrid wireless network deployments.

We define \( A = \pi(R/2)^2 \) to be the area of a disk centered at each MANET node, and say two nodes are connected if their disks touch (or overlap) one another. It is well known that a phase transition occurs in the connectivity of such a MANET-only network (i.e., a giant connected component appears) when the dimensionless node-density parameter \( \eta \equiv \lambda A \) is just above the threshold \( \eta_c \approx 1.28 \). The phase-transition can be simulated efficiently using an algorithm developed by Mertens and Moore [10] (which is an extension of an algorithm developed by Newman and Ziff [11]) that runs essentially in \( O(n) \) time. We choose the connectivity range \( R = 1 \) for our simulations, and use the Mertens-Moore algorithm to evaluate the size of the largest connected component as a function of the normalized node density \( \eta \). The algorithm simulates continuum site percolation by generating an instance of a rate-\( \lambda \) Poisson deployment of \( n \) nodes indexed \( \{1, \ldots, n\} \) over an \( L^2 \)-area square region, and thereafter ‘occupying’ one node at a time as per a pre-determined random permutation \( \Pi(j) \), \( 1 \leq j \leq n \). When the node \( \Pi(j) \) is occupied, the algorithm adds new edges between \( \Pi(j) \) and all its already-occupied one-hop neighbors, and merges connected components (clusters) at each step using a union-find algorithm, i.e., by pointing the marker node of one cluster (every cluster has a unique marker node) to that of another one.

We begin with a MANET-only deployment with \( L = 10 \) (area, \( L^2 = 100 \)). We consider two cases, \( \lambda_1 = 1.0186 \), and \( \lambda_2 = 1.8 \). Note that \( \lambda_1 < \lambda_c < \lambda_2 \), where the critical node density \( \lambda_c = \eta_c/(\pi R^2/4) \approx 1.426 \), for \( R = 1 \). The actual number of nodes generated in our simulations were \( n_1 = 109 \) and \( n_2 = 180 \). For each case, we choose one root node and three terminal nodes at random. We compute the multicast cost for the respective choices of root and terminal nodes using Algorithm 2. All the physical cost parameters used for the simulations are given in Table I. We then randomly choose 4 MANET nodes at a time, and convert them to hybrid nodes. The cellular connectivity is determined using an underlying base-station network with base-stations located at the centers of the faces of a honeycomb lattice. After each round of MANET to hybrid conversion, we re-evaluate the connectivity (largest component size as a function of \( \eta \)), and compute the multicast cost using Algorithm 2 after adding all the nodes. We hold the permutation order of node addition (\( \Pi \)) in the Mertens-Moore algorithm same for each run, so as to minimize statistical variability. Our results are summarized in Fig. 9. We see that when \( \lambda = \lambda_2 \) (super-critical regime), adding hybrid capability to nodes does not increase the largest cluster size (since the network is already connected), but progressively reduces the multicast cost. When \( \lambda = \lambda_1 \) (sub-critical regime), adding hybrid capability to nodes both increases the size of the largest cluster as well as reduces multicast cost. An analytical characterization of the downward shift of the continuum percolation threshold with a given density of hybrid nodes, and its scaling with respect to multicast cost—both for Algorithms 2 and 3, remains subjects of future work.
In this paper, we have assumed that the cellular links are point-to-point (P-to-P) because that is the current configuration in which telecom operators operate their networks. However, the 3GPP and LTE standards have proposed support for point-to-multipoint (P-to-M) links anticipating the increasing popularity of voice and video multicast in future [1]. In the proposed Multimedia Broadcast Multicast Services (MBMS) [6], it is possible for a base station node (BS) to communicate either in the P-to-P mode or in the P-to-M mode to reach one or more mobiles in its own cell. These cellular P-to-M links can be incorporated into our model and technique proposed in Section III without much difficulty in the following manner.

First of all, MBMS considers the situation where the content provider is on an Internet server and the content does not originate on a mobile device—the case we have considered in this paper, as this scenario can be easily addressed by adding a P-to-P uplink from a mobile to a content server node (CS) and a P-to-M downlink from S to all the terminals in a particular cell. The aforementioned style of P-to-M cellular transmission can be modeled in our hybrid network multicast setup by making the following modification.

Secondly, instead of all mobiles with type C/H in the hybrid network directly forming a cellular clique with edge weights (with no explicit BS nodes), we now add a base station node \( B_{S_k} \) in each cell \( k \), an uplink from each of the C/H nodes in the cell \( k \) to \( B_{S_k} \), and a directed hyperedge from \( B_{S_k} \) to all the C/H nodes in cell \( k \). These BS nodes are assigned type BS in addition to the currently existing nodes of types C, M, or H. Now, instead of the C/H nodes forming a clique, only the BS nodes form a clique among themselves (with very low edge costs) and the C/H nodes are only connected to their nearest BS nodes in a star topology (considering only cellular links). The uplinks and P-to-P downlinks get costs \( \sim \beta \cdot d(u, B_{S_k})^{-\alpha} \); however, the P-to-M downlinks (which are now hyperedges) get a different cost which is a function of the largest distance between a BS and a mobile in that corresponding cell. This is analogous to the MANET broadcast cost which is a function of the “transmission range”. Note that even the BS nodes may be sources of content if the content is originating somewhere in the wide area Internet.

It is easy to see that our directed hypergraph construction approach in Sections III-A and III-B and the algorithm in Section IV-A are general enough to incorporate the above modifications to the cellular network connectivity. However, algorithm in Section IV-B needs more significant modifications. This is a topic of future research.

We note that the broadcast nature of the cellular BS has subtly different ramifications from the broadcast nature of MANET. In MANET every node can broadcast, whereas in Cellular only BS nodes can broadcast, and thus two adjacent cells in cellular can only be connected by Internet connections between corresponding BS nodes unlike in MANET where two neighborhoods can be bridged directly (and wirelessly) by intermediate MANET nodes.

In this paper, we have investigated the problem of multicasting in hybrid wireless networks possessing both point-to-point and broadcast links. We have modeled the problem with directed hypergraphs and multigraphs and gave two distinct approximation algorithms. We have shown using numerical simulations that the algorithm based on hypergraphs has a better approximation factor but a much higher time complexity than the algorithm based on multigraphs. Hence the multigraph algorithm may in practice be a feasible choice. It is also amenable to a simpler distributed implementation for dynamic networks, which we plan to investigate in the future. Using algorithmic techniques from percolation theory, we have also showed that incremental support of hybrid capability to nodes of a MANET-only network progressively reduces the critical node density at which long-range connectivity appears, as well as simultaneously reducing the multicast cost, both when the original MANET-only network is in the sub-critical and super-critical connectivity regimes. We plan to analytically study phase transitions and trade-off in connectivity and multicast cost using the two algorithms proposed. We also intend to explore applications to real deployment scenarios in our future work.

**References**


